

# SECOND LAW ANALYSIS OF WASTE HEAT RECOVERY HEAT EXCHANGER USING SUPERCRITICAL CARBON DIOXIDE

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## Abstract

The purpose of this study is analysis of second law of the helical coil heat exchanger for research in the field of waste heat recovery. By the second law analysis we can combine both the effects of heat transfer and pressure drop in a single equation to know the exact irreversibilities occurring in the system. The performance parameters used for the analysis are entropy generation number and second law efficiency. The parametric study has been carried out to know the behaviour of the systems by varying the tube diameter, length, mass flow rate of SCO<sub>2</sub> (supercritical carbon dioxide) and inlet temperature of SCO<sub>2</sub>. The second law efficiency or rational efficiency is having a maximum value at optimum diameter which is 0.015 m; it decreases with mass flow rate but increase with inlet temperature of SCO<sub>2</sub>. Entropy generation number is also having minimum value at optimum diameter which is 0.015 m. It increases with mass flow rate of SCO<sub>2</sub> but decreases with inlet temperature of SCO<sub>2</sub>.

**Keywords:** waste heat recovery, supercritical carbon dioxide, second law analysis, parametric analysis, Entropy generation minimization.

## 1. Introduction

Basic power cycles dispose of an extensive segment of useful energy into the environment by means of exhaust gases. Using supercritical power cycle, this wasted energy might be used for power generation and production of hot water. Supercritical power cycle utilizes a supercritical working fluid for maximum effectiveness of heat exchanger. Carbon dioxide is chosen as the working fluid since it has a generally low critical

temperature which makes it alluring for medium temperature waste heat applications

**Chen** et al. [1] simulated the performance of a CO<sub>2</sub> power cycle in utilizing the low-grade heat sources and the results was analysed with a focus on second law thermodynamics. **Dostalet al.** [2] performed parametric optimization of supercritical CO<sub>2</sub> Brayton power cycles for application to advanced nuclear reactors. **Sarkar** [3] conducted exergetic analyses and parametric optimization of S-CO<sub>2</sub> recompression cycle. **Rao et al.** [4] provided heat transfers and pressure drop characteristics of heat exchangers using SCO<sub>2</sub> as working fluid based on many parameters such as size, mass flux, inlet temperature and pressure, etc. **Wang et al.** [5] carried out the heat transfer characteristics of supercritical CO<sub>2</sub> cooled in the helically coiled tube. **Prabhanjan** et al. [6] determined advantages of using a helically coiled heat exchanger instead of straight tube heat exchanger for heating liquids. **Coronelet al.** [7] determined pressure drop and friction factor (f) correlations in helical coil heat exchangers under turbulent flow conditions based on the Reynolds number, curvature ratio, and temperature was developed. **Andhare et al.** [8] investigated convective heat transfer coefficients of a helical coil heat exchanger experimentally. **Manjunath et al.** [9] conducted second law analysis of unbalanced heat exchangers by varying L/D ratio for parallel and counter flow configurations.

A program code is established with help of EES software for calculation required for the waste heat recovery analysis and variation of operating parameter such as length, diameter of the helical coil and temperature, mass flow rate of supercritical carbon dioxide at inlet condition. The effect of these parameters on system performance are investigated. By the second law analysis we are able to combine both the effects of heat transfer and pressure drop in a single close form equation by which we are able to know the exact irreversibility occurring in the system which can be used to improve the performance.

### Nomenclature

R	Shell radius (m)
$L_m$	Tube bundle length (m)
A	tube radius(m)
D	Tube diameter (m)
P	Pitch (m)
Th	Tube thickness (m)
T	Temperature (K)
$\dot{m}$	Mass flow rate (Kg/s)
P	Pressure (kPa)
Q	Heat duty (W)
$C_p$	Specific heat (kJ/kg-K)
$\mu$	Viscosity (Pa-s)
$\rho$	Density (kg/m <sup>3</sup> )
K	Conductivity (W/m-K)
N	Number of tube
$Re$	Reynold number
Pr	Prandtl number
$R_{eff}$	Radius of curvature (m)
H	Film coefficient (W/m <sup>2</sup> -K)
U	Overall heat transfer coefficient (W/m <sup>2</sup> -K)
$A_s$	Surface area (m <sup>2</sup> )
$N_b$	Number of rotation of tube bundle
F	Friction factor
$\delta P$	Pressure drop (Pa)
Hg	Hagen number
$n_{tr}$	Number of effective tube bundle
E	Effectiveness of heat exchanger
$\Psi$	Rational efficiency
$\dot{E}$	Exergy (W)
$I$	Irreversibility (W)
$N_s$	Overall entropy generation number
$N_{SH}$	Entropy generation number because of

	heat transfer
$N_{Sp}$	Entropy generation number by virtue of pressure drop
$S_{gen,H}$	Entropy generation because of heat transfer (W/K)
$S_{gen,P}$	Entropy generation by the virtue of pressure drop (W/K)
NTU	Number of transfer unit
EES	Engineering equation solver

### Subscripts

Ex	Exhaust gas in shell side
sc <sub>CO2</sub>	Super critical carbon dioxide in tube side
I	Inner
O	Outer
Tub	Turbulent
Lam	Laminar
Avg	Average

## 2. Analysis

In this section of heat exchanger's thermal design and Second law are analysed. By the second law analysis we are able to combine both the effects of heat transfer and pressure drop in a single close form equation by which we are able to know the exact irreversibility occurring in the system which can be used to improve the performance.

### 2.1 Thermal Design of Heat Exchanger

In this type of heat exchanger, the number of tubes ( $N_{scCO2}$ ) in the bundle can be determined as follows[10]

$$N_{scCO2} = (R_{out} - R_{in}) * \frac{L_m}{p^2} \quad (1)$$

#### A. Flow Parameter:

Velocity of Supercritical Carbon dioxide is given as [10]

$$Vel_{scCO2} = \frac{m_{scCO2}}{\rho_{scCO2} * \frac{\pi}{4} * d_i^2 * N_{scCO2}} \quad (2)$$

Reynold number in tube side is given as [10]

$$Re_{sco2} = \frac{\rho_{sco2} \cdot Vel_{sco2} \cdot d_i}{\mu_{sco2}} \quad (3)$$

Reynold number for exhaust gas in shell side [10]

$$Re_{ex} = \frac{\rho_{ex} \cdot Vel_{ex} \cdot d_o}{\mu_{ex}} \quad (4)$$

Velocity for exhaust gas in shell side [10]

$$Vel_{sco2} = \frac{m_{sco2}}{\rho_{ex} \cdot \pi \cdot (R_{out}^2 - R_{in}^2) \cdot \left[1 - \left(\frac{d_o}{p}\right)\right]} \quad (5)$$

Prandtl number in the shell side [10]

$$Pr_{ex} = \frac{\mu_{ex} \cdot c_{p_{ex}}}{k_{ex}} \quad (6)$$

Prandtl number in the tube side [10]

$$Pr_{sco2} = \frac{\mu_{sco2} \cdot c_{p_{sco2}}}{k_{sco2}} \quad (7)$$

### B. Heat Transfer Correlations:

In this part, heat transfer for the helical coiled heat exchanger will be estimated.

#### Tube Side:

In the tube side, heat transfer correlations are based on the heat transfer in the helical coiled tubes. To estimate it, some geometrical features should be defined first as follows:

Tube Radius [10]

$$a = \frac{d_i}{2} \quad (8)$$

Radius of Curvature: [10]

$$R_{eff} = \frac{R_{in} + R_{out}}{2} \quad (9)$$

Based on that, Nusselt number for helical coiled tube can be estimated as follows Shah et al. [13]:  
 Nusselt number for straight pipe [10]

$$Nusselt_0 = 0.022 \cdot Re_{sco2}^{0.8} \cdot Pr_{sco2}^{0.5} \quad (10)$$

Nusselt number for helical coil, Shah et al. [11]

$$Nusselt_{sco2} = Nusselt_0 \cdot \left[1.0 + 3.6 \cdot \left[1 - \left(\frac{a}{R_{eff}}\right)\right] \cdot \left(\frac{a}{R_{pitch}}\right)^{0.8}\right] \quad (11)$$

heat transfer coefficient in the tube side can be given as follows: [10]

$$h_{sco2} = Nusselt_{sco2} \cdot \frac{k_{sco2}}{d_i} \quad (12)$$

#### Shell side:

Nusselt number for shell, zukauskas [12]

$$Nusselt_{ex} = 0.27 \cdot Re_{ex}^{0.63} \cdot Pr_{ex}^{0.36} \quad (13)$$

heat transfer coefficient in the tube side can be given as:

$$h_{ex} = Nusselt_{ex} \cdot \frac{k_{ex}}{d_o} \quad (14)$$

Overall heat transfer:

In this part, the overall heat transfer coefficient for the helical coiled heat exchanger is estimated. Effect of heat transfer resistance at the wall has been neglected because of small tube thickness.

Overall heat transfer coefficient: [10]

$$U = \frac{1}{\frac{1}{h_{sco2}} + \frac{1}{h_{ex}}} \quad (15)$$

Average temperature difference: [10]

$$\Delta T_{avg} = T_{ex} - T_{sco2} \quad (16)$$

Heat transfer Surface Area: [10]

$$A_s = \frac{Q}{U \cdot \Delta T_{avg}} \quad (17)$$

Tube length: [10]

$$L_{t_{middle}} = \frac{A_s}{\pi \cdot \left(\frac{d_i + d_o}{2}\right) \cdot N_{sco2}} \quad (18)$$

Number of rotation of tube bundle: [10]

$$N_b = \frac{L_{t_{middle}}}{\pi \cdot (R_{out} + R_{in})} \quad (19)$$

### C. Pressure Drop

In this part, pressure drop in shell and the tubes are estimated for the helical coiled heat exchanger:

#### Tube Side:

Pressure drop in tube side can be given by Kakac and Liu [13]:

Friction factor in helical coiled Tubes: [13]

$$f = \left[0.0084 \left[Re_t \cdot \left[\frac{(R_{in} + R_{out})}{a}\right]^{-2}\right]^{-0.2} \cdot \left[\frac{(R_{in} + R_{out})}{a}\right]^{-0.5}\right] \quad (20)$$

Pressure drop in helical tube side: [13]

$$\Delta P_{sco2} = 4 \cdot f \cdot \frac{L_{t_{middle}}}{d_i} \cdot \rho_{sco2} \cdot \frac{Vel_{sco2}^2}{2} \quad (21)$$

#### Shell Side:

Pressure drop in the tube bundles in cross flow can be given by Martin, Shah & Sekulic [14]:

$$Pr_{ratio} = \frac{p}{d_o} \quad (22)$$

For inline tube bundles, Hagen number is given by, Martin [14]:

$$H_{g_{iam}} = \frac{140 \cdot Re_{ex} \cdot (Pr_{ratio}^{0.5} - 0.6)^2 + 0.75}{\left[Pr_{ratio}^{1.6} \cdot \left(\frac{4 \cdot Pr_{ratio}^2}{\pi} - 1\right)\right]} \quad (23)$$

$$H_{g_{tub}} = \left[ \left[0.11 + \frac{0.6 \cdot \left(1 - \frac{0.94}{Pr_{ratio}}\right)^{0.6}}{(Pr_{ratio} - 0.85)^{1.3}}\right] \cdot 10^{0.47 \cdot (-0.5)} + 0.015 \cdot (Pr_{ratio} - 1)^2 \right] \cdot Re_{ex}^{1.9} \quad (24)$$

Total Hagen number is given as

$$H_g = H_{g_{iam}} + H_{g_{tub}} \quad (25)$$

Number of effective tube bundles:

$$n_{tr} = N_D * \left(\frac{L_m}{p}\right) \quad (26)$$

Pressure drop in shell side:

$$\delta P_{ex} = \frac{\mu_{ex}^2 * n_{tr} * Hg}{\rho_{ex} * d_s^5} \quad (27)$$

## 2.2 Second law efficiency analysis of heat exchanger

Number of transfer unit –NTU is defined as

$$NTU = \frac{U * A_s}{C_{min}} \quad (28)$$

Min heat capacity is of exhaust gas ( $C_h$ )

$$C_{min} = cp_{ex} * \dot{m}_{ex} \quad (29)$$

Max heat capacity is of sco2 ( $C_c$ )

$$C_{max} = cp_{sco2} * \dot{m}_{sco2} \quad (30)$$

Ratio of  $C_{min}$  to  $C_{max}$  is C which is given as

$$C = \frac{C_{min}}{C_{max}} \quad (31)$$

Assuming flow in helical coil heat exchanger as counter flow. The effectiveness is given as [11]

$$\varepsilon = \frac{1 - \exp(-NTU * (1 - C))}{1 - C * \exp(-NTU * (1 - C))} \quad (32)$$

The second law efficiency which is the ratio of desired exergy output to exergy used. Kotas [15]

Rational efficiency or Exergetic efficiency:

$$\psi = \frac{\dot{E}_{desiredoutput}}{\dot{E}_{used}} \quad (33)$$

The  $\dot{E}_{desiredoutput}$  is the sum of all exergy output from the system while  $\dot{E}_{used}$  is the exergy input into system and irreversibilities are the exergy destruction as

$$\dot{E}_{used} = \dot{E}_{desiredoutput} + \dot{I} \quad (34)$$

$\dot{E}_{desiredoutput}$  Can be expressed in terms of effectiveness and inlet temperature and

$$\dot{E}_{desiredoutput} = C_{max} * \left[ \varepsilon * C * (T_{ex} - T_{sco2}) - T_o * \ln \left\{ 1 + \left( \frac{\varepsilon * C * (T_{ex} - T_{sco2})}{T_{sco2}} \right) \right\} \right]$$

	Exhaust Gas 100% load	Exhaust Gas 71% load	Exhaust Gas 49% load	Supercritical CO <sub>2</sub>
T (K)	843	798	761	308
$\dot{m}$ (Kg/s)	63	55	48	35
P (KPa)	200	170	140	20000

$$(35)$$

Irreversibility is the product of the reference temperature and entropy generation rate.

Irreversibility given by  $\dot{I}$

$$\dot{I} = T_o * N_s * C_{min} = T_o * \dot{S}_{gen} \quad (36)$$

Entropy generation number is defined as the ratio of entropy generation to minimum heat capacityrate, i.e.,  $C_{min}$ [10]

Entropy generation number

$$N_s = N_{sH} + N_{sP} \quad (37)$$

Entropy generation number are given as

Entropy generation number because of heat transfer

$$N_{sH} = \ln \left[ 1 + \varepsilon * \left( \frac{T_{ex}}{T_{sco2}} - 1 \right) \right] + \left[ \left( \frac{\varepsilon}{C} \right) * \ln \left\{ 1 - C * \varepsilon * \left( 1 - \frac{T_{sco2}}{T_{ex}} \right) \right\} \right] \quad (38)$$

Entropy generation number because of pressure drop

$$N_{sP} = \left( \frac{R_{sco2}}{cp_{sco2} * C} \right) * \left( \frac{\delta P_{sco2}}{P_{sco2}} \right) + \left( \frac{R_{ex}}{cp_{ex}} \right) * \left( \frac{\delta P_{ex}}{P_{ex}} \right) \quad (39)$$

## 3. Results and Discussion

The basic input parameters which are taken from typical marine gas turbine[16], are used for the heat exchanger design and are summarized below in the table 1 to 3. In this heat exchanger cold fluid is supercritical carbon dioxide which is flowing in helical tube and hot fluid is exhaust gas which is flowing in shell side.

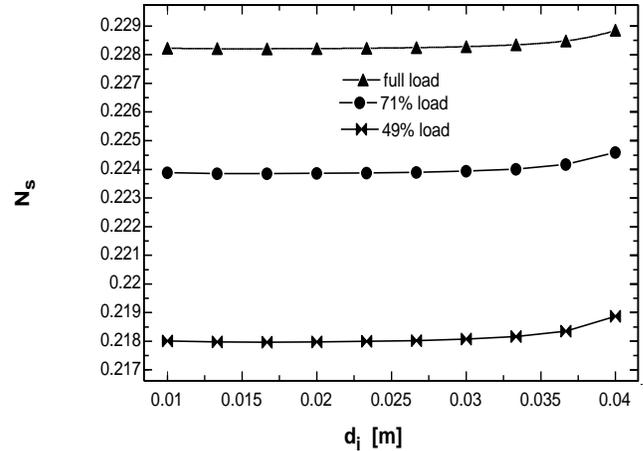
Table 1: Heat Exchanger Input Geometric Parameters

R <sub>out</sub> (m)	R <sub>in</sub> (m)	L <sub>m</sub> (m)	d <sub>i</sub> (m)	P (m)	th (m)
3.5	1.5	1.0	0.03	0.05	0.001754

Table 2: Heat Exchanger Input Operating Parameter for Exhaust Gas and SCO<sub>2</sub>

*Table 3: Properties of Exhaust gas and SCO<sub>2</sub> at various load*

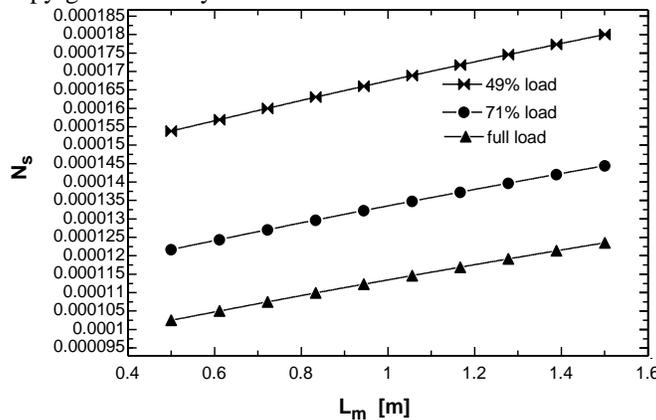
	Exhaust Gas 100% load	Exhaust Gas 71% load	Exhaust Gas 49% load	Supercritical CO <sub>2</sub>
P (Kg/m <sup>3</sup> )	0.8266	0.7422	0.6409	866.4
K (W/m-K)	0.0594	0.05705	0.05506	0.09851
C <sub>p</sub> (kJ/kg-K)	1.108	1.098	1.090	2.2
$\mu$ (kg/m-s)	0.00003763	0.00003635	0.00003528	0.00008368



*Fig 1 Overall Entropy Generation Number of Heat Exchanger VS tube inner diameter*

Fig 1 demonstrate overall entropy generation number of heat exchanger at various load verses tube inner diameter. Initially overall entropy generation number is decreasing with increase in tube inner diameter and obtains a minimum value at 0.015 m diameter and after that it is increasing with increase in tube inner diameter. Initially when diameter increases then pressure drop decreases and entropy generation by virtue

of pressure drop decreases because it dominates here. After getting minimum value of overall entropy generation number it starts increasing because entropy generation because of heat transfer will increase and dominate with increment in diameter. Here optimum diameter can be achieved by second law analysis which can't be obtained by first law thermodynamics analysis.



*Fig 2 Entropy Generation Number due to Pressure Drop VS Tube bundle length*

Fig 2 illustrate entropy generation number because of pressure drop verses tube bundle length. Entropy generation number by virtue of pressure drop is increasing with tube bundle

length. As length increases pressure drop will increase so entropy generation number by virtue of pressure drop will increase. So overall entropy generation will increase.

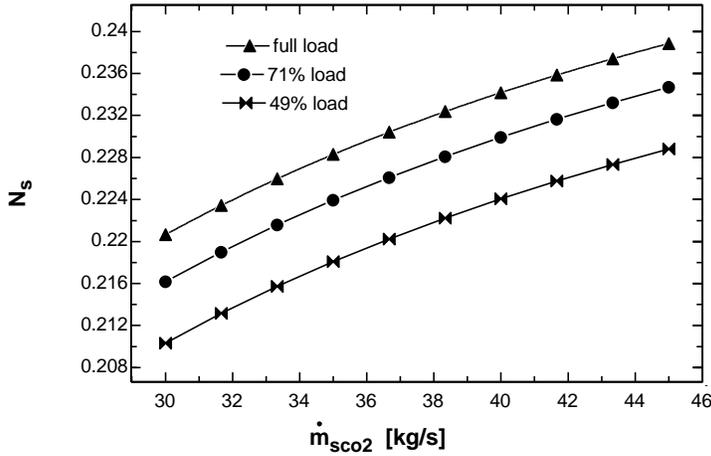


Fig 3 Overall Entropy Generation Number of Heat Exchanger VS SCO2 mass flow rate

Fig 3 illustrates Overall entropy generation number of heat exchanger at various load versus SCO2 mass flow rate. Overall entropy generation number increases with increase in SCO2 mass flow rate. This is due to as mass

flow rate increases, pressure drop also increases, due to this entropy generation because of pressure drop will increase which dominates entropy generation as a result of heat transfer therefore overall entropy generation number increases with increase in SCO2 mass flow rate.

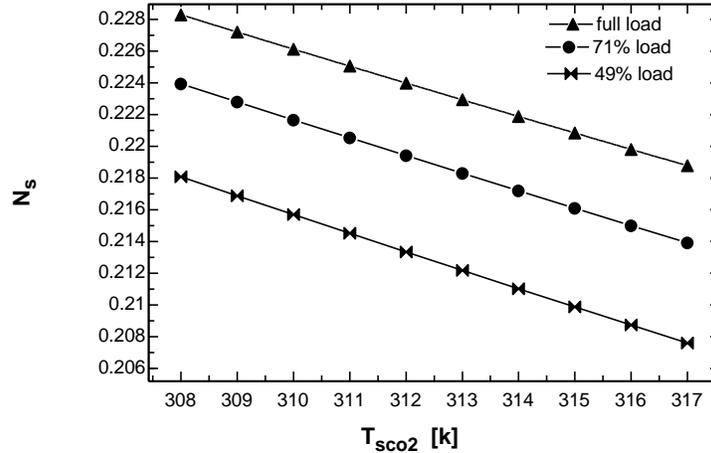
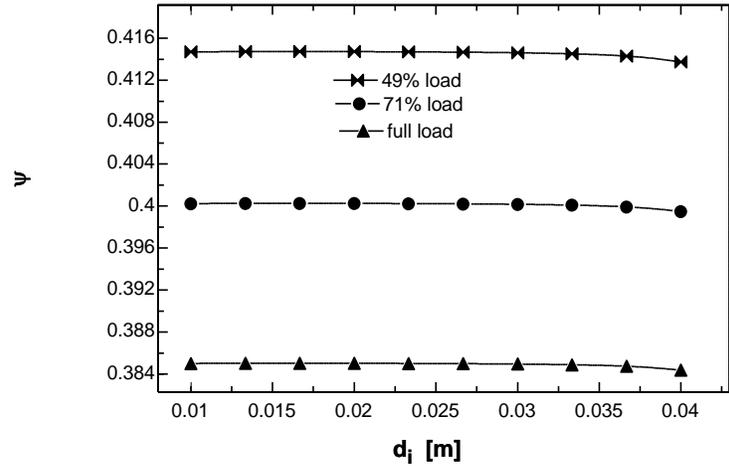


Fig 4 Overall Entropy Generation Number of Heat Exchanger VS SCO2 inlet temperature

Fig 4 demonstrate Overall entropy generation number of heat exchanger versus SCO2 inlet temperature. Overall entropy generation number is decreasing with increase in SCO2 inlet temperature. As temperature of cold fluid is

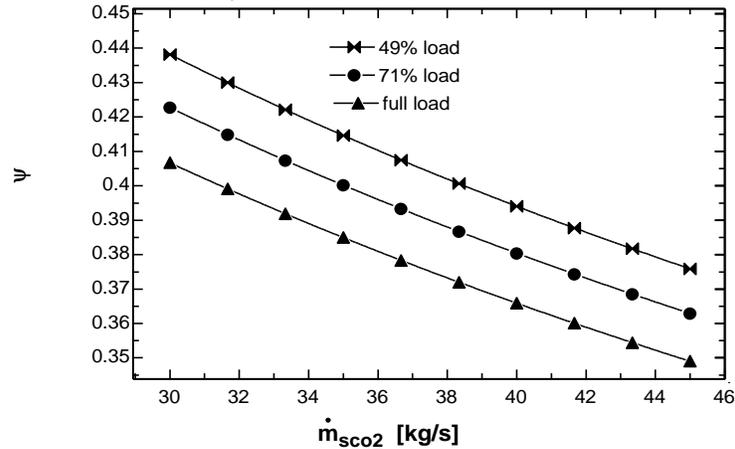
increasing, temperature difference between two fluids is decreasing so entropy generation because of heat transfer will decrease and it will dominate entropy generation by virtue of pressure drop therefore overall entropy generation number will decreasing with increase in SCO2 inlet temperature.



*Fig 5 Rational Efficiency of Heat Exchanger at various load VS tube inner diameter*

Fig 5 portrays Rational Efficiency of Heat Exchanger at various load versus tube inner diameter. Rational Efficiency of Heat Exchanger is increasing with increase in tube inner diameter, becomes maximum at 0.015 m diameter and then decreases. Initially when

diameter increases then pressure drop decreases so entropy generation by virtue of pressure drop will decrease and it dominates here so rational Efficiency increases. It starts decreasing because entropy generation as a result of to heat transfer will increase.



*Fig 6 Rational Efficiency of Heat Exchanger at various load VS SCO2 mass flow rate*

Fig 6 illustrates Rational Efficiency of Heat Exchanger at various load versus SCO2 mass flow rate. Rational Efficiency is decreasing with increase in SCO2 mass flow rate. This is as a

result of to as mass flow rate increases, pressure drop also increases, due to this entropy generation because of pressure drop also increases therefore rational efficiency will decrease.

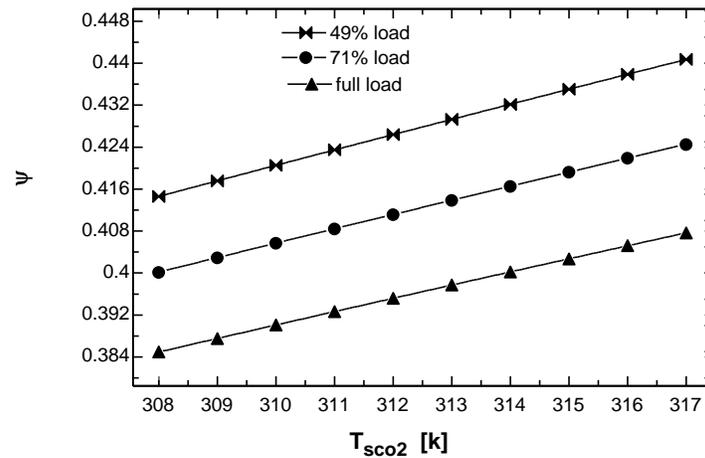


Fig 7 Rational Efficiency of Heat Exchanger at various load VS SCO2 inlet temperature

Fig 7 demonstrates Rational Efficiency of Heat Exchanger at various load versus SCO2 inlet temperature. Rational Efficiency of Heat Exchanger is increasing with increase in SCO2 inlet temperature. As temperature of cold fluid is increasing, temperature difference between two fluids is decreasing so entropy generation as a result to heat transfer will decrease therefore rational efficiency will increase.

#### 4. Conclusions

From analysis of second law, the following are some of the conclusions to improve the overall performance of the system. The second law efficiency or rational efficiency is having a maximum value for varying the diameter of the tube, which results in optimum value of the diameter at 0.015 m. As mass flow rate increases, second law efficiency decreases, but it increases as inlet temperature of SCO2 increases. In entropy generation number behaviour also, we are able to obtain minimum value for varying the diameter and the value obtained is 0.015 m corresponding to lower irreversibility. As the mass flow rate of SCO2 increases, entropy generation number increases but entropy generation number decreases for increase in inlet temperature of SCO2. These results are useful for heat exchanger's thermal design based on entropy generation minimization. Here optimum diameter can be achieved by second law analysis which can't be obtained easily by first law thermodynamics analysis. Second law of

thermodynamic combines entropy generation by virtue of heat transfer and pressure drop which is not possible from first law of thermodynamics.

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